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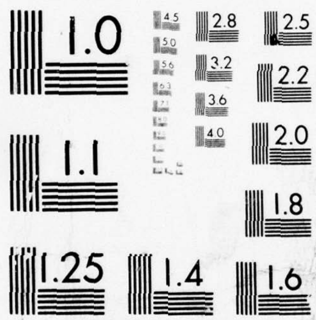
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NEW YORK UNIVERSITY

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Department of Meteorology and Oceanography
Geophysical Sciences Laboratory, Report No. 63-13

Part XVI

A THREE DIMENSIONAL MODEL OF THE WIND DRIVEN HORIZONTAL VELOCITIES IN THE NORTH ATLANTIC OCEAN (VI)

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and

Frank D. Malone

The research reported in this document has been sponsored by the U. S. Naval Oceanographic Office, Washington, D. C. under Contract No. N62306-794.

October 1963

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A THREE DIMENSIONAL MODEL OF THE WIND DRIVEN HORIZONTAL VELOCITIES IN THE NORTH ATLANTIC OCEAN. (VI).

Part VI. Miscellanea.
(The Effect of the Convergence of the Meridians on the Westward Intensification of the Oceanic Circulation).
(Approximations of the Nonlinear Terms in the Vertically Integrated Equation of Motion).

10 El Sayed Mohamed Hassan

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Frank D. Malone

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The Effect of the Convergence of the Meridianson the Westward Intensification of the Oceanic Circulation

Abstract:

Models for a wind driven ocean were studied including the effects of sphericity and rotation of the earth. Like Stommel's 1948 model, they exhibited the westward intensification of the circulation, but significant difference in pattern appeared.

Introduction:

Stommel (1948) established that the variation of the Coriolis parameter with latitude is responsible for the westward intensification in an ocean of homogeneous water. In that model, he assumed that the variation is uniform with latitude. A study was initiated to investigate the effects of dropping that assumption. Further, the inclusion of the Coriolis parameter variation, makes rectangular ocean models bounded by meridians and latitude circles on a rotating earth inconsistent, as it can only change where meridians converge. Two further models were therefore considered, where the meridians converge. One modelled a conical earth, consistent with a unform change in the Coriolis parameter, and the other modelled a spherical earth.

Equations:

The treatment was kept as near to Stommel's 1948 treatment as possible, and most of the original symbols are thus retained. Assume an ocean bounded by two meridians and two latitude circules with corners at $(0,0)$, $(\lambda, 0)$, (λ, b), $(0, b)$. Assume further that a zonal wind stress steadily blows over the ocean and is expressed by $\tau_x = -F \cos\frac{\pi y}{b}$.

The equations of motion thus are:

$$f\,(D+h)\,v\rho - F\cos(\pi y/b) - Ku\,(D+h) - g\rho(D+h)\,\frac{\partial h}{\partial x} = 0 \tag{1}$$

$$-f\,(D+h)\,u\rho \qquad\qquad - Kv\,(D+h) - g\rho(D+h)\,\frac{\partial h}{\partial y} = 0 \tag{2}$$

The equation of continuity is:

(a) for the case where the convergence of the meridians is ignored

$$\frac{\partial}{\partial x}\Big((D+h)\,u\Big) + \frac{\partial}{\partial y}\Big((D+h)\,v\Big) = 0 \tag{3a}$$

(b) for the conical earth

$$\frac{\partial}{\partial x}\Big((D+h)\,u\Big) + \frac{\partial}{\partial y}\Big((D+h)\,v\Big) = \frac{1}{y_o - y}\Big(v\,(D+h)\Big) \tag{3b}$$

(c) for the case of a spherical earth

$$\frac{\partial}{\partial x}\Big(u\,(D+h)\Big) + \frac{\partial}{\partial y}\Big(v\,(D+h)\Big) = \frac{\tan\varphi}{R}\Big(v\,(D+h)\Big) \tag{3c}$$

Equations 3a, 3b, 3c allow the introduction of a stream function Ψ such that

$$u\,(D+h) = -\frac{\partial \Psi}{\partial y}\ , \qquad v\,(D+h) = +\frac{\partial \Psi}{\partial x} \tag{4}$$

Utilizing this relation a vorticity equation can be developed

$$K\,\nabla^2\Psi + \rho\beta\frac{\partial \Psi}{\partial x} = -\frac{\partial \tau_x}{\partial y} \tag{5a}$$

$$K\,\nabla^2\Psi + \rho\beta\frac{\partial \Psi}{\partial x} - \frac{K}{y_o - y}\,\frac{\partial \Psi}{\partial y} = -\frac{\partial \tau_x}{\partial y} + \frac{\tau_x}{y_o - y} \tag{5b}$$

$$K\,\nabla^2\Psi + \frac{2\Omega\rho\cos\varphi}{R}\,\frac{\partial \Psi}{\partial x} - \frac{K\tan\varphi}{R}\,\frac{\partial \Psi}{\partial y} = -\frac{\partial \tau_x}{\partial y} + \frac{\tan\varphi}{R}\,\tau_x \tag{5c}$$

where

b the length of the ocean along a meridian

f the Coriolis parameter

g the acceleration of gravity

h the displacement of the sea surface from its undisturbed level

u the velocity in the zonal direction

v the velocity in the meridional direction

x the zonal coordinate, increasing eastward

y the meridional coordinate, increasing northward

y_o the coordinate of the pole in the conical earth

D the depth of the ocean

F the maximum wind stress

K the coefficient of friction

R the radius of the earth

β the variation of Coriolis parameter with latitude

∇ $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

λ the maximum width of the ocean considered

ρ the density of the water

τ the wind stress $\equiv F \cos \dfrac{\pi y}{b}$

φ the latitude

Ψ the transport stream function

Equations (5a), (5b), (5c) were solved numerically and the solutions appear in Figure 1, 2, 3, 4. Figure 1 reproduces Stommel's picture, while 2 is for the same model with a variable β. Figures 2 and 4 are for a conical and for a spherical earth respectively. Dimensions of the oceans and relevant parameters appear in Table I. Latitudes are given on all of the figures, though, they are relevant only in Figures 2 and 4.

Table I: Dimensions of oceans and other relevant parameters.

b	=	length of ocean along a meridian = 0.5×10^9 cm
λ	=	maximum width of ocean = 10^9 cm
y_o	=	the coordinate of the pole in the conical earth = 10^9 cm
D	=	the depth of the ocean = 200 m
F	=	maximum wind stress = 1.0 dyne cm^{-2}
K	=	coefficient of friction = 10^{-6} dynes cm^{-4} sec
R	=	radius of earth = 0.6371×10^9 cm
β	=	(when used as a constant) = 10^{-13} cm^{-1} sec^{-1}
ρ	=	density of water = 1 gm cm^{-3}

Discussion of the Results:

The circulation remains intensified in the west, but the pattern shows interesting changes. When the convergence of the meridians were ignored, the center of the gyre was displaced to the north. The transport decreased as compared with the case of a constant β. It is to be noted, however, that the mean (when β varied) was greater than the constant value that was used corroborating the results of a study made by Garner

(1962) about the relation between the value of β and the transport.

When the convergence of the meridians were taken into account, however, the center of the transport gyre was displaced to the south. The value of the transport was again decreased, reflecting, in addition to the β effect, the effect of the decrease in area affected by the wind. A weak circulation with the opposition sense of rotation also appeared at the northwestern corner of the model.

References:

Garner, David, M., 1962: Some Studies on the Ocean Circulation, New York University. Technical Report prepared for ONR Contract No. NONR 285-(03).

Stommel, Henry, 1948: The Westward Intensification of Wind-Driven Ocean Currents. Transactions, American Geophysical Union, Vol. 29, No. 2.

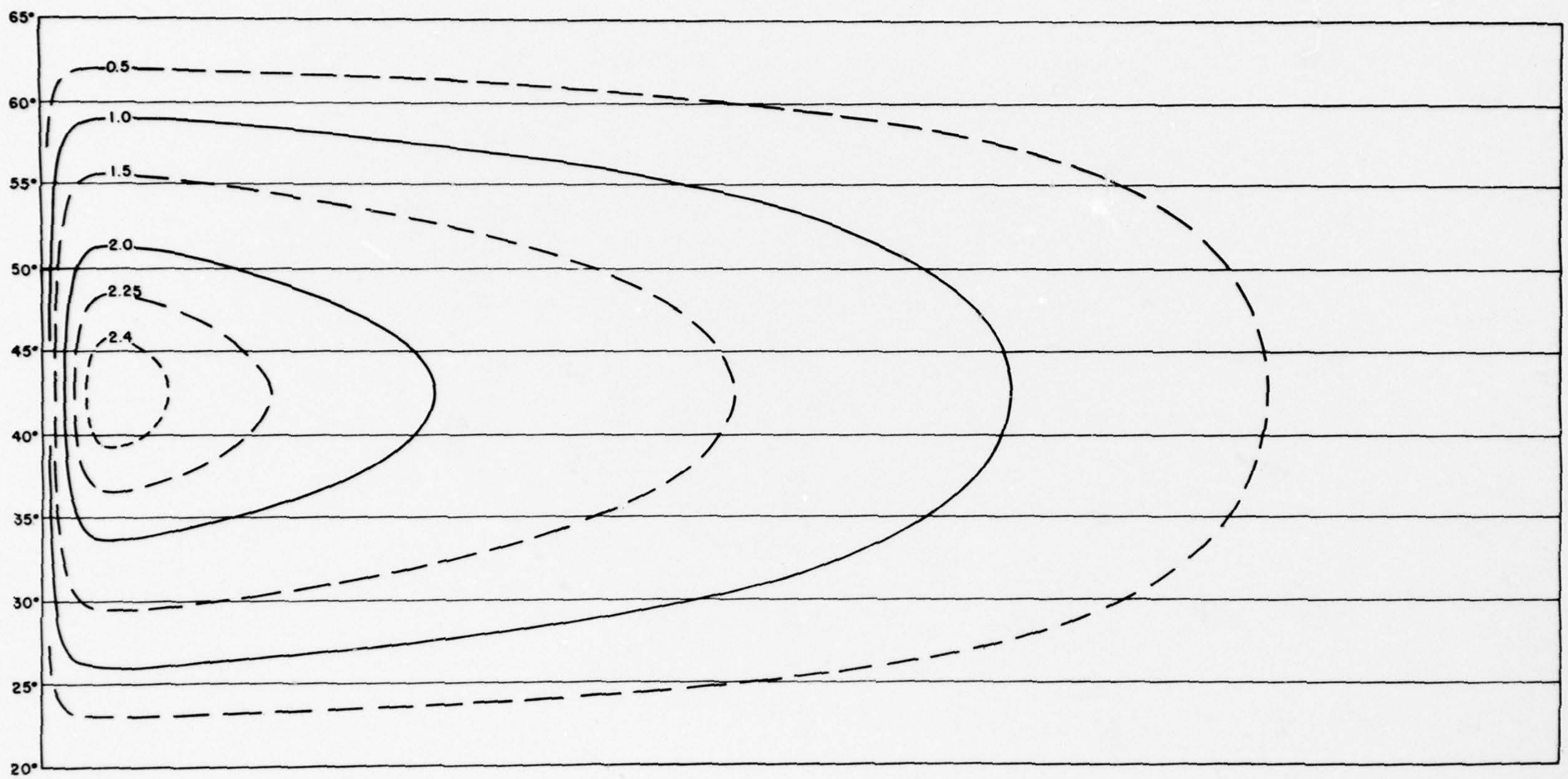

Figure 1. Stommel's model, rectangular ocean and constant β. Stream functions in c.g.s. $\times 10^{+9}$.

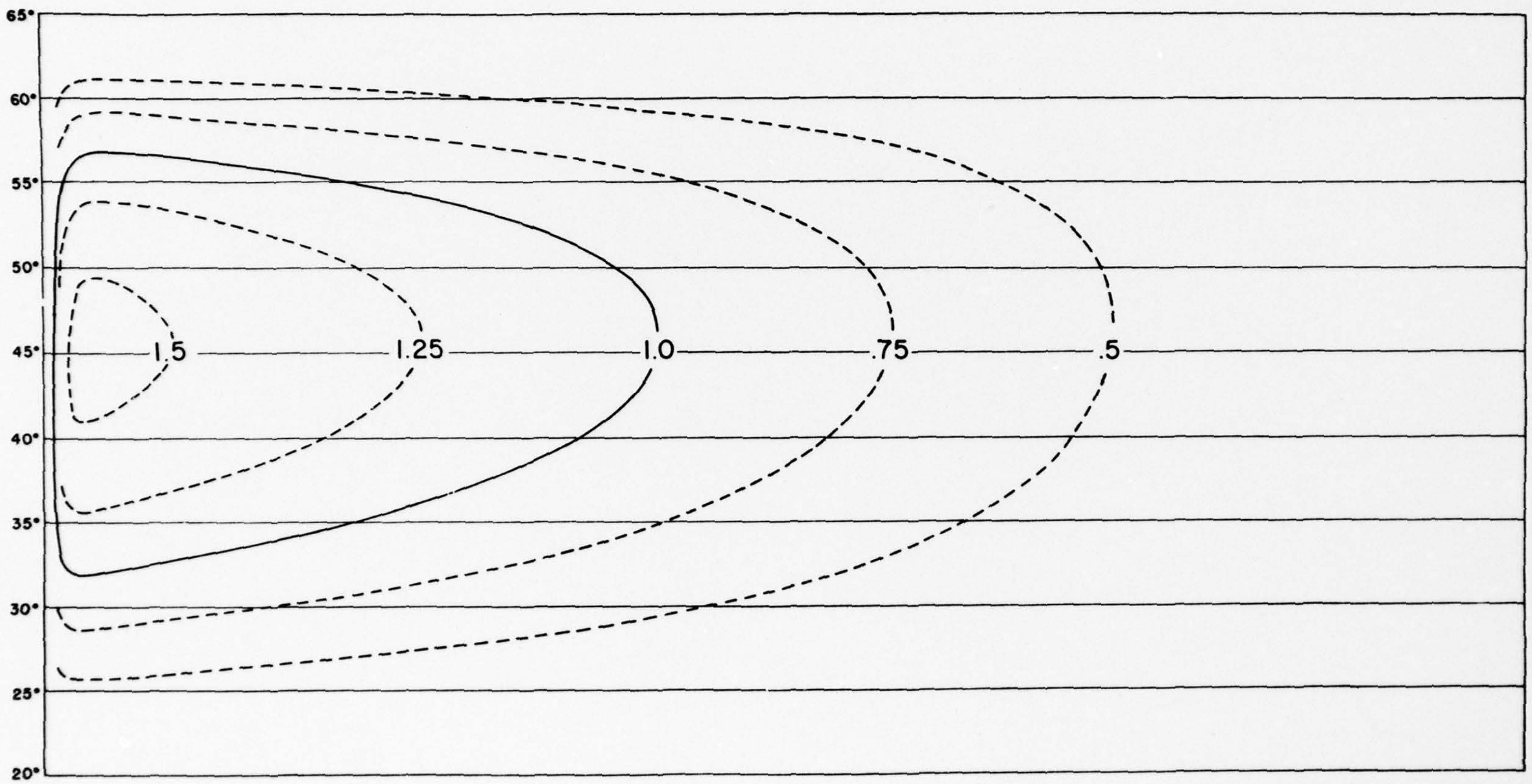

Figure 2. First modified model, rectangular ocean and variable β. Stream functions in c.g.s. $\times 10^{+9}$.

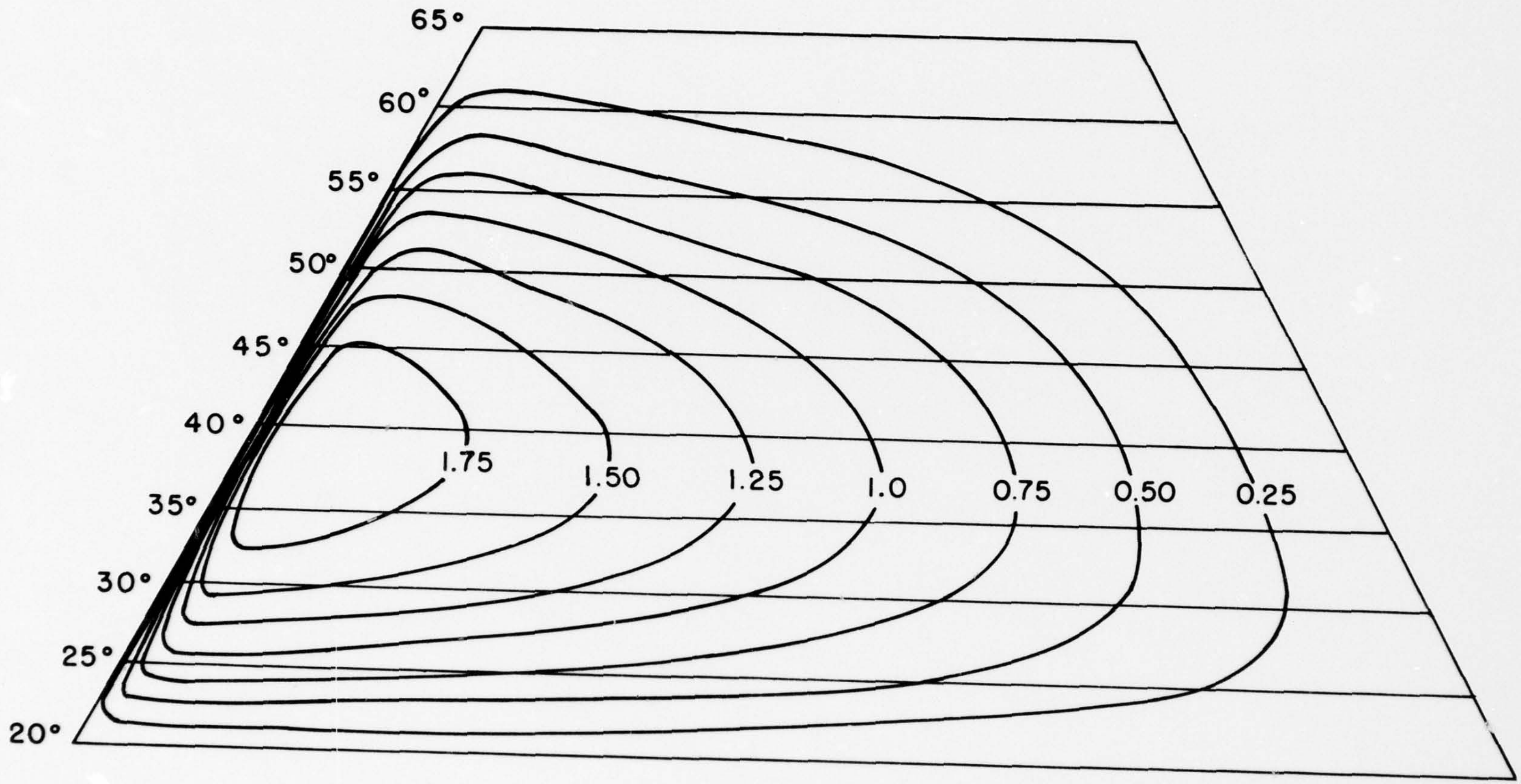

Figure 3. Second modified model, conical earth, constant β. Stream functions in c.g.s. $\times 10^{+9}$.

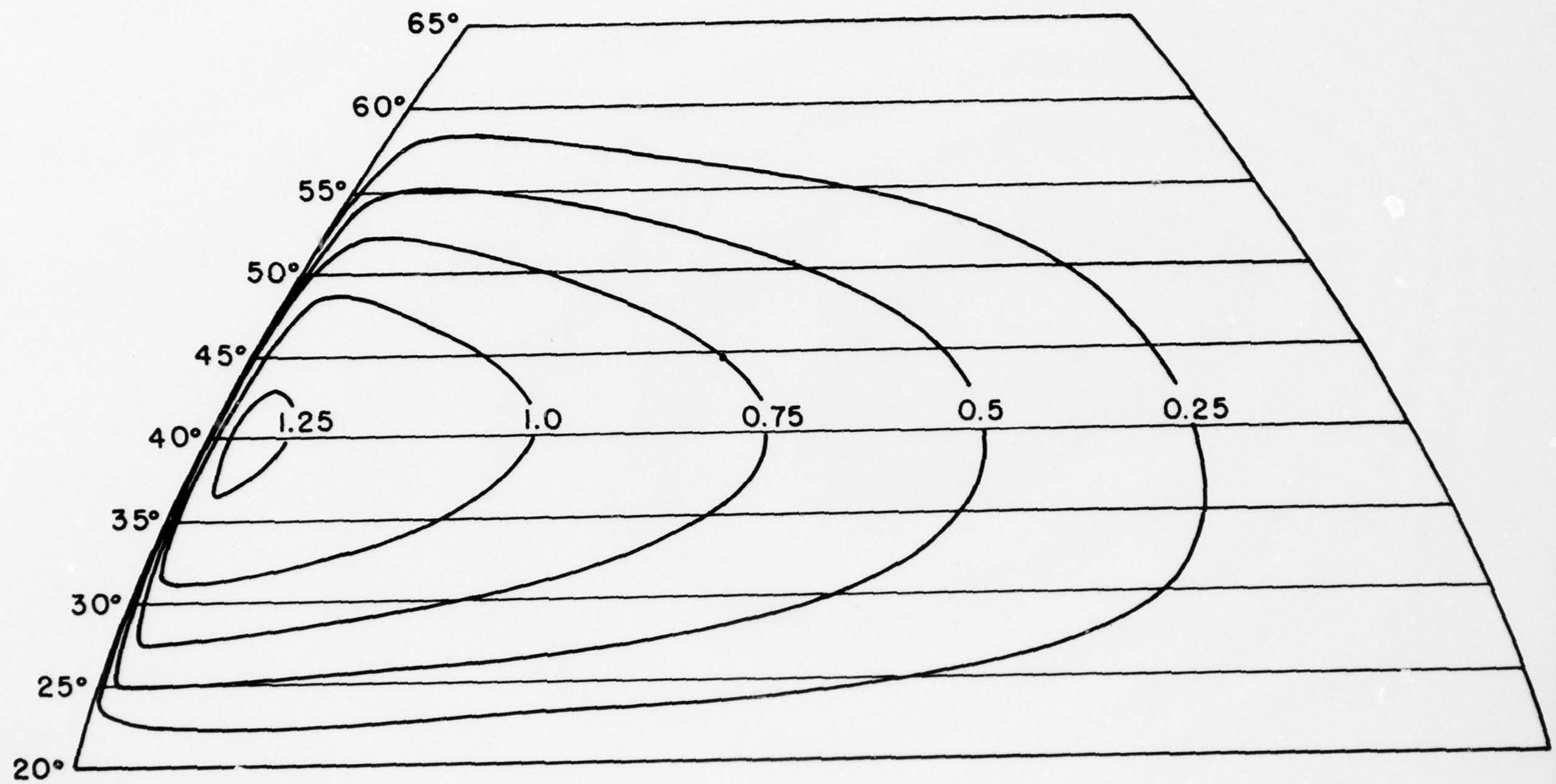

Figure 4. Third modified model, spherical earth, variable β. Stream functions in c.g.s. × 10^{+9}.

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Approximations of the Nonlinear Terms in the Vertically Integrated Equation of Motion

Abstract:

Integrated nonlinear terms in the hydrodynamical equations of motion were compared with the nonlinear terms formed from the integrated velocity. This showed a difference in form to indicate an essential difference between the two expressions. Numerical values of the two expressions were compared for a particular model and the results substantiated the theoretical treatment.

Introduction:

The nonlinear terms in the hydrodynamical equations of motion pose a challenge when the equations are considered for solution. For a variety of reasons, the equations become more amenable to treatment when they are integrated vertically. This poses an additional difficulty as there is no simple expression relating the vertically integrated nonlinear terms with those which can be formed from the vertically integrated velocities. It has been occasionally assumed, however, (Bryan 1963, Carrier and Robinson 1962) that a simple proportionality exists. This is investigated here.

The velocity is expanded in the vertical in a Fourier series in such a way that the velocity at the bottom automatically vanishes without imposing any restrictions on the velocity profile. Expressions are then evaluated for the integrated nonlinear terms and the nonlinear terms formed from the integrated velocities. These terms are then compared.

Procedure:

Assume that $$u = \sum_{i=1}^{\infty} u_i \cos \frac{(2i-1)\pi z}{2h}$$

Then $$\frac{\partial u}{\partial x} = \sum_{i=1}^{\infty} \frac{\partial u_i}{\partial x} \cdot \cos \frac{(2i-1)\pi z}{2h}$$ (assuming that h does not vary with x)

and $$u \frac{\partial u}{\partial x} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} u_i \frac{\partial u_j}{\partial x} \left(\cos \frac{(2i-1)\pi z}{2h} \cos \frac{(2j-1)\pi z}{2h} \right)$$

$$= \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j \neq i}^{\infty} u_i \frac{\partial u_j}{\partial x} \left(\cos \frac{i+j-1)\pi z}{2h} + \cos \frac{(i-j)\pi z}{2h} \right)$$

$$+ \frac{1}{2} \sum_{i=1}^{\infty} u_i \frac{\partial u_i}{\partial x} \left(\cos \frac{(2i-1)\pi z}{h} + 1 \right)$$

and $$\int_0^h u \frac{\partial u}{\partial x} dz = \frac{h}{2} \sum_{i=1}^{\infty} u_i \frac{\partial u_i}{\partial x} \qquad (1)$$

while $$\int_0^h u \, dz = \sum_{i=1}^{\infty} \frac{2h}{(2i-1)\pi} u_i \left[\sin \frac{(2i-1)\pi z}{2h} \right]_0^h$$

$$= \sum_{i=1}^{\infty} \frac{2h}{(2i-1)\pi} u_i (-1)^{i+1}$$

and $$\frac{\partial}{\partial x} \int_0^h u \, dz = \sum_{i=1}^{\infty} \frac{2h}{(2i-1)\pi} \frac{\partial u_i}{\partial x} (-1)^{i+1}$$

$$\text{Thus} \int_0^h u\,dz \cdot \frac{\partial}{\partial x}\int_0^h u\,dz = \sum_{i=1}^{\infty}\sum_{j\neq i}^{\infty} \frac{4h^2}{(2i-1)(2j-1)\pi^2}\, u_i \frac{\partial u_j}{\partial x}(-1)^{i+j}$$

$$+ \sum_{i=1}^{\infty} \frac{4h^2}{(2i-1)^2\pi^2}\, u_i \frac{\partial u_i}{\partial x} \qquad (2)$$

It is obvious that (1) and (2) do not have a simple proportionality factor. To get a further feeling for these terms, an oceanic model was used, for which an analytic solution could be produced (Hassan, 1963). The model has many deficiencies, the most relevant here is that it does not contain the nonlinear terms. The terms computed were the following:

$$\int_0^h fu\,dz,\ \int_0^h fv\,dz,\ \int_0^h u\frac{\partial u}{\partial x}\,dz,\ \int_0^h u\frac{\partial v}{\partial x}\,dz,\ \int_0^h v\frac{\partial u}{\partial y}\,dz,\ \int_0^h v\frac{\partial v}{\partial y}\,dz,$$

$$\frac{1}{h}\int_0^h u\,dz\,\frac{\partial}{\partial x}\int_0^h u\,dz,\ \frac{1}{h}\int_0^h u\,dz\,\frac{\partial}{\partial x}\int_0^h v\,dz,\ \frac{1}{h}\int_0^h v\,dz\,\frac{\partial}{\partial y}\int_0^h u\,dz,$$

$$\frac{1}{h}\int_0^h v\,dz\,\frac{\partial}{\partial y}\int_0^h v\,dz.$$

The first two terms were computed to indicate whether the model can still be used to evaluate the nonlinear terms, even though the nonlinear terms were not considered in the solution. Not surprisingly, except for the extreme western boundary, the nonlinear terms were negligible compared to the terms retained, and only at the western boundary, were they of comparable magnitude (Figure 1 and 2). The two expressions

for the nonlinear terms showed very little relationship to one another, as can be seen from Figures 3 to 10.

Conclusion:

Vertically integrated nonlinear terms cannot be approximated by expressions using the vertically integrated velocities.

References:

Bryan, K., 1963: A numerical investigation of a nonlinear model of a wind driven ocean. Personal communications.

Carrier, G. F. and A. Robinson, 1962: On the theory of the wind driven ocean circulation. Journal of Fluid Mechanics, Vol. 12, Part 1, pp. 49-80.

Hassan, E. M., (in press): The General Oceanic Circulation: Some Computations, A Working Hypothesis and Proposed Tests.

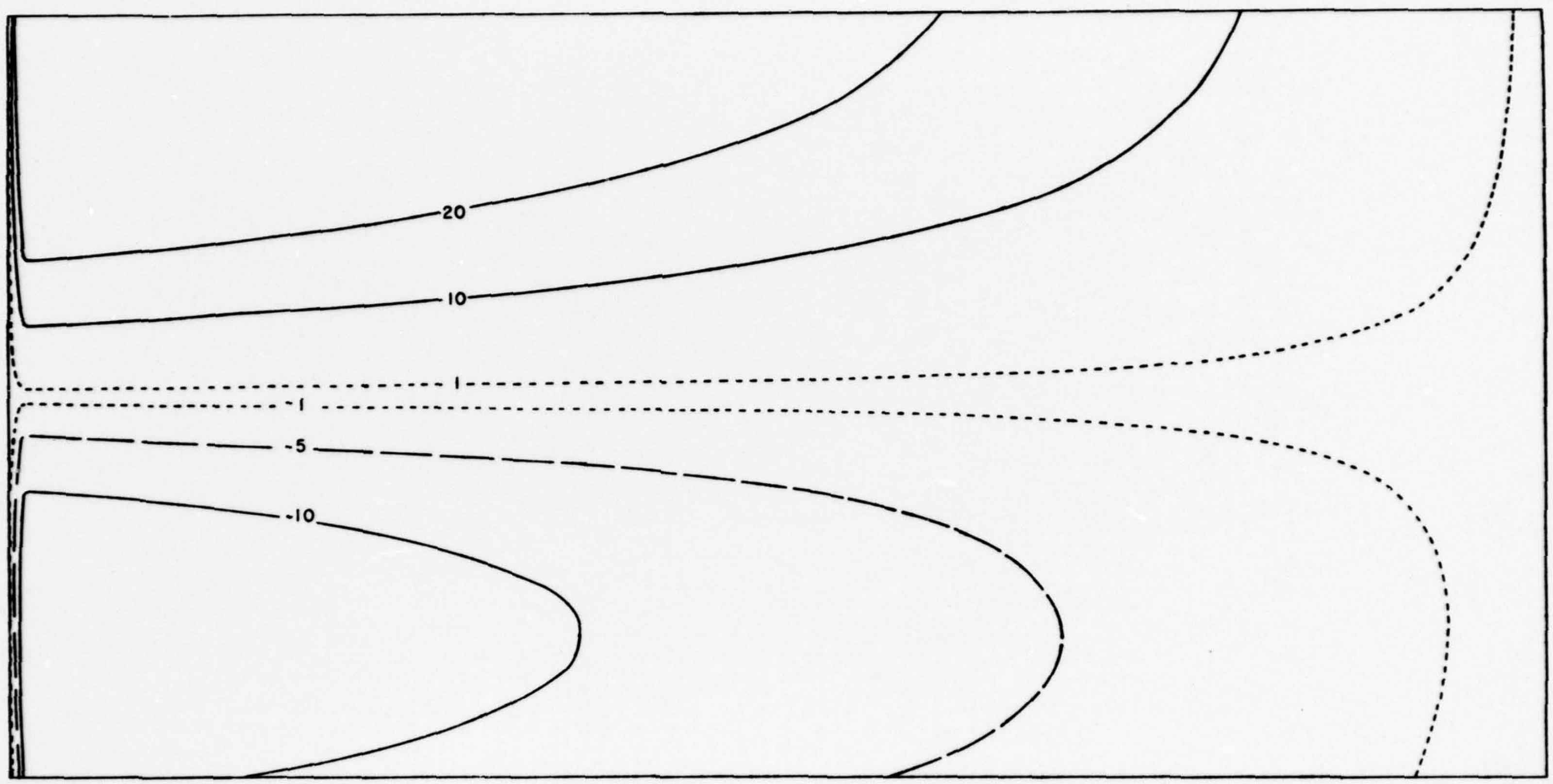

Figure 1. $f\int_{o}^{h} u\,dz$ in c.g.s. units.

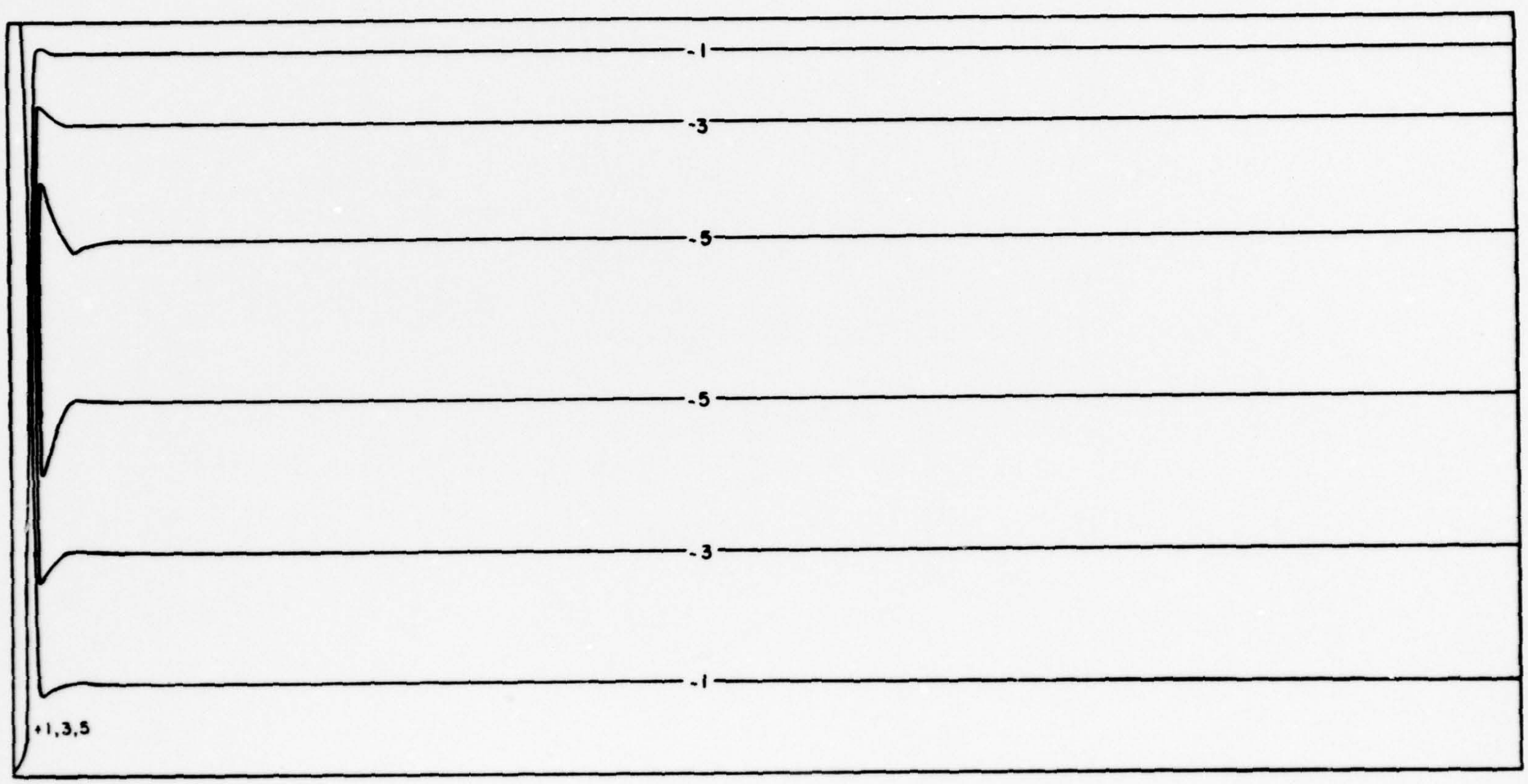

Figure 2. $f\int_0^h v\,dz$ in c.g.s. units.

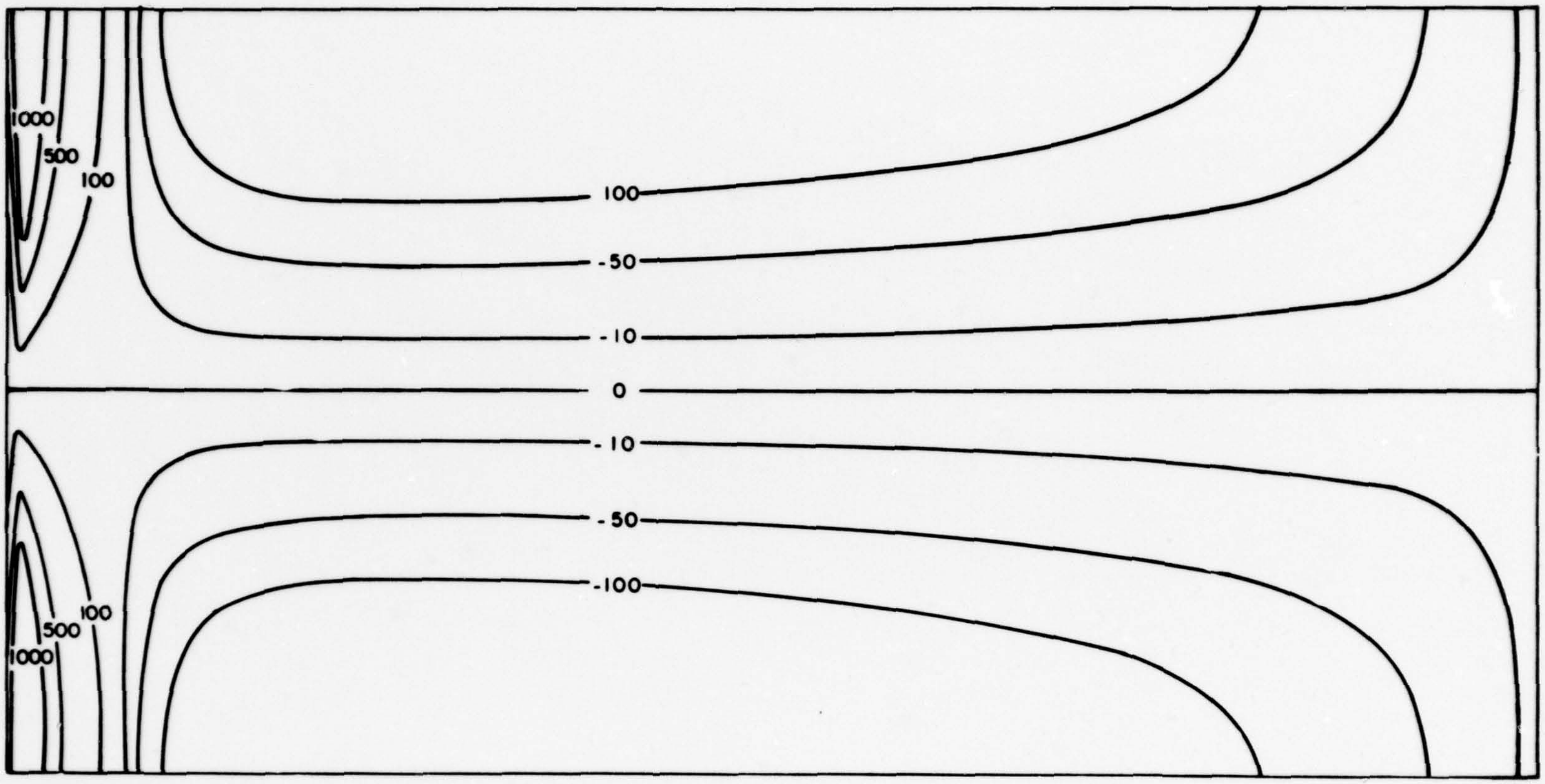

Figure 3. $\int_{o}^{h} u \frac{\partial u}{\partial x} \, dz$ in c.g.s. units $\times 10^{-3}$.

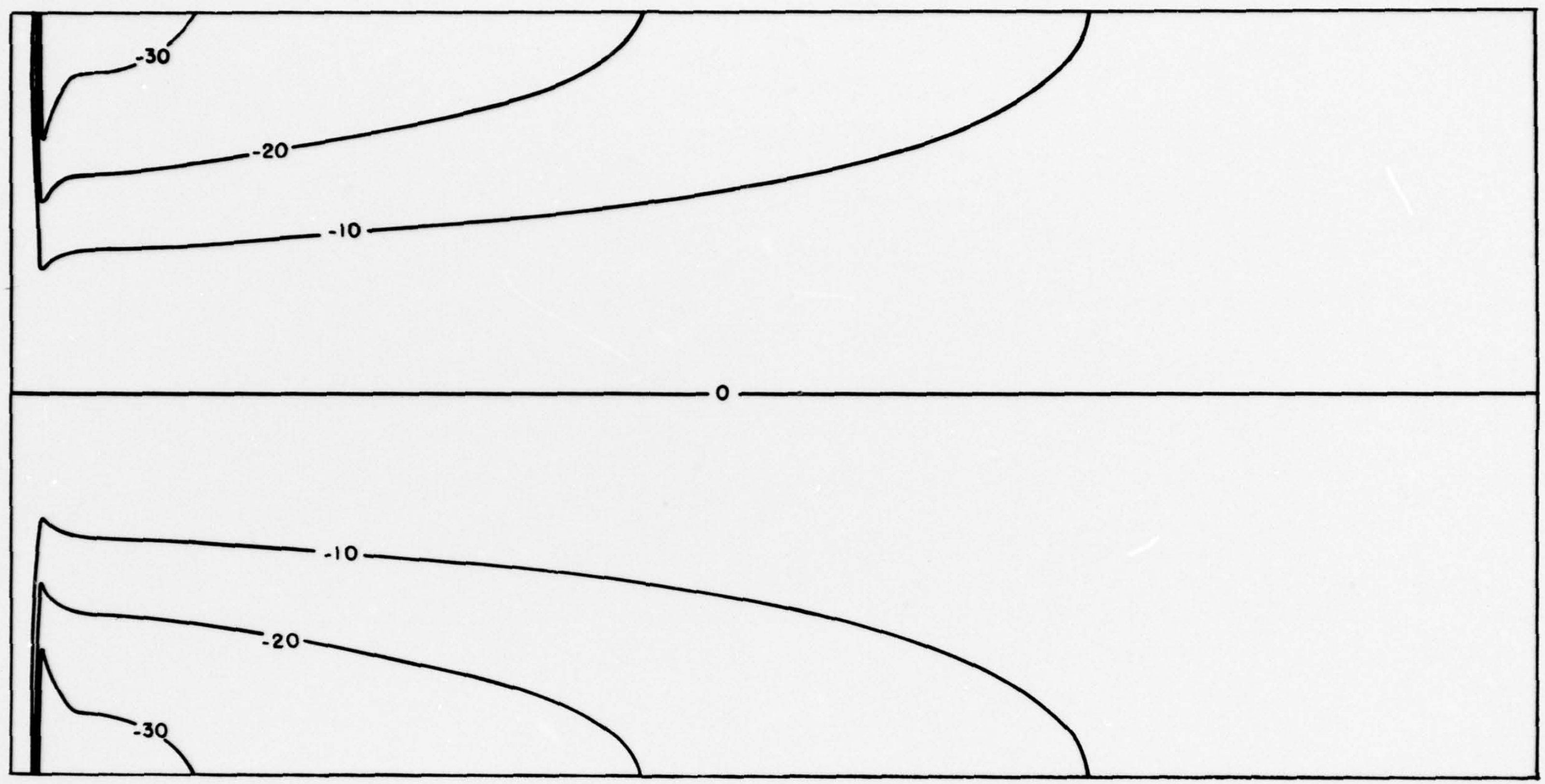

Figure 4. $\frac{1}{h}\int_{o}^{h} u\,dz \cdot \frac{\partial}{\partial x}\int_{o}^{h} u\,dz$ in c.g.s. units $\times 10^{-3}$.

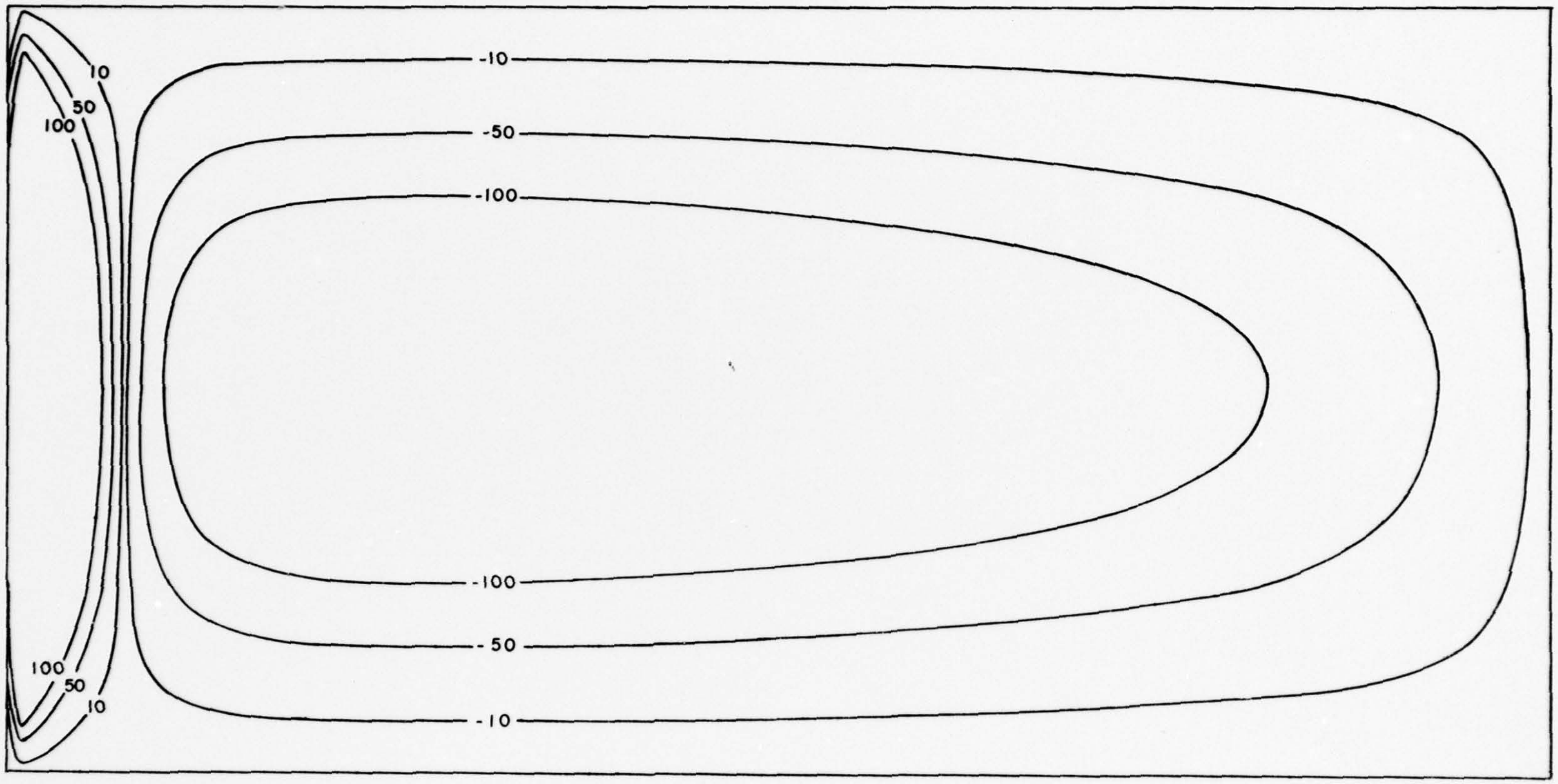

Figure 5. $\int_0^h v \frac{\partial u}{\partial y} dz$ in c.g.s. units $\times 10^{-4}$.

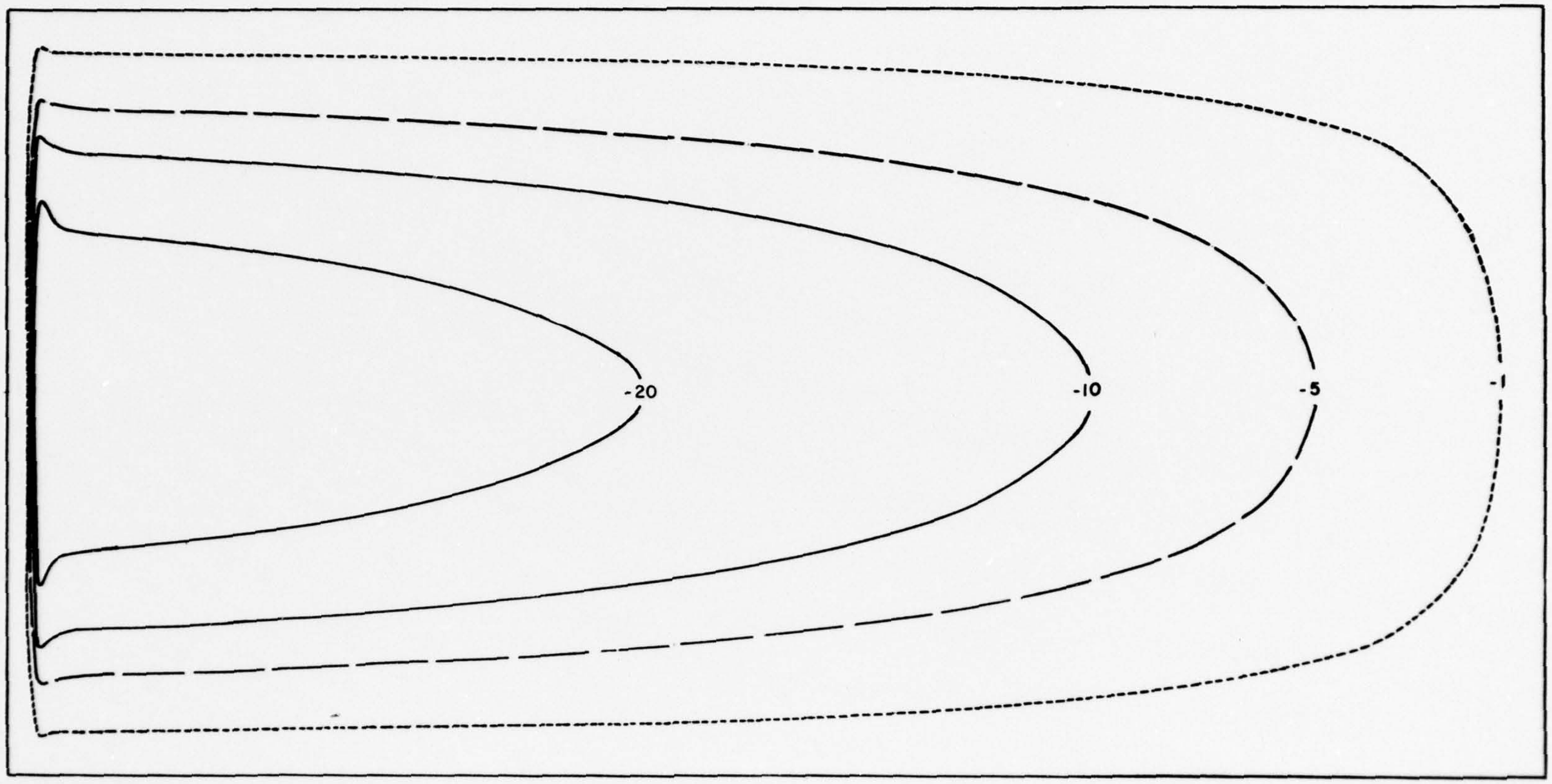

Figure 6. $\frac{1}{h}\int_{o}^{h} v\,dz \cdot \frac{\partial}{\partial y}\int_{o}^{h} u\,dz$ in c.g.s. units $\times 10^{-4}$

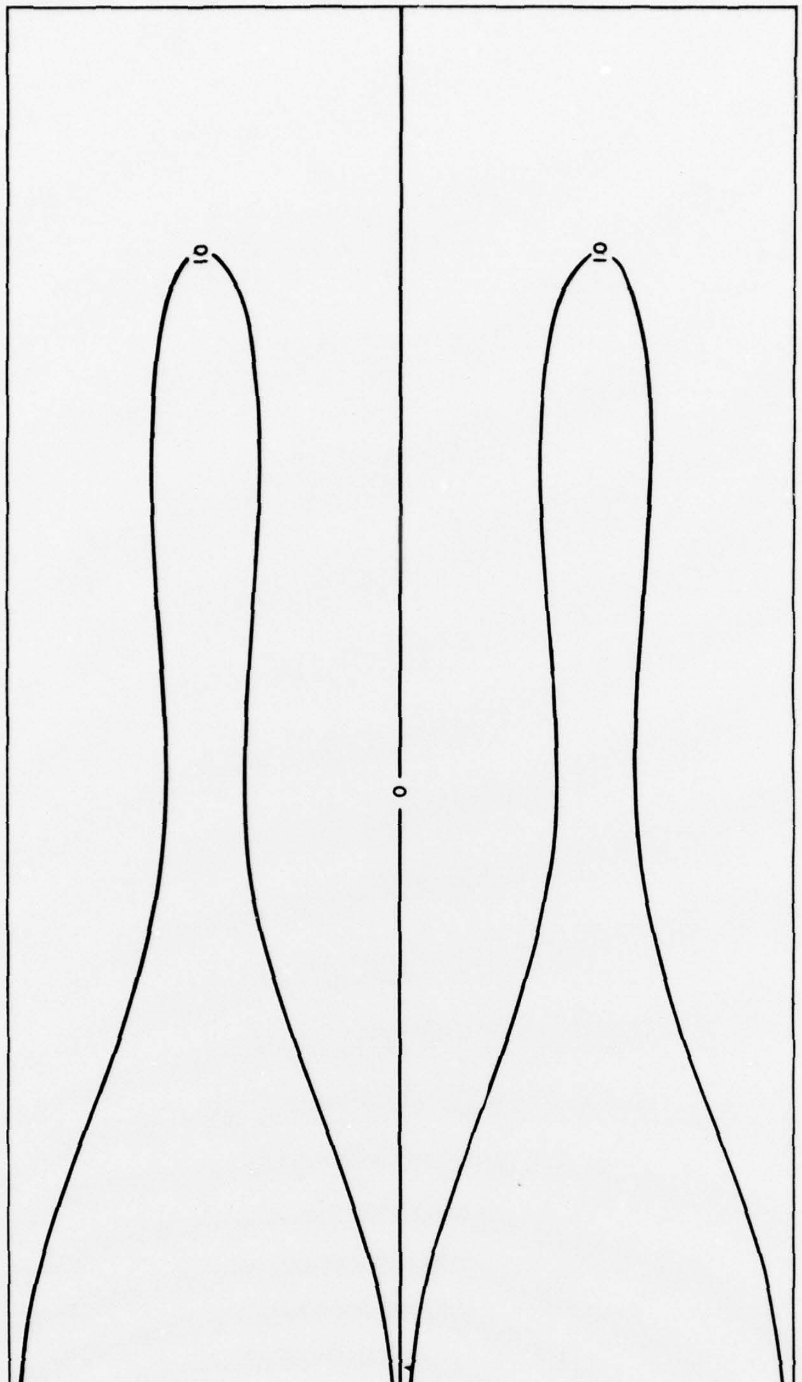

Figure 7. $\int_0^h u \frac{\partial v}{\partial x} dz$ in c.g.s. units x 10^{-4}.

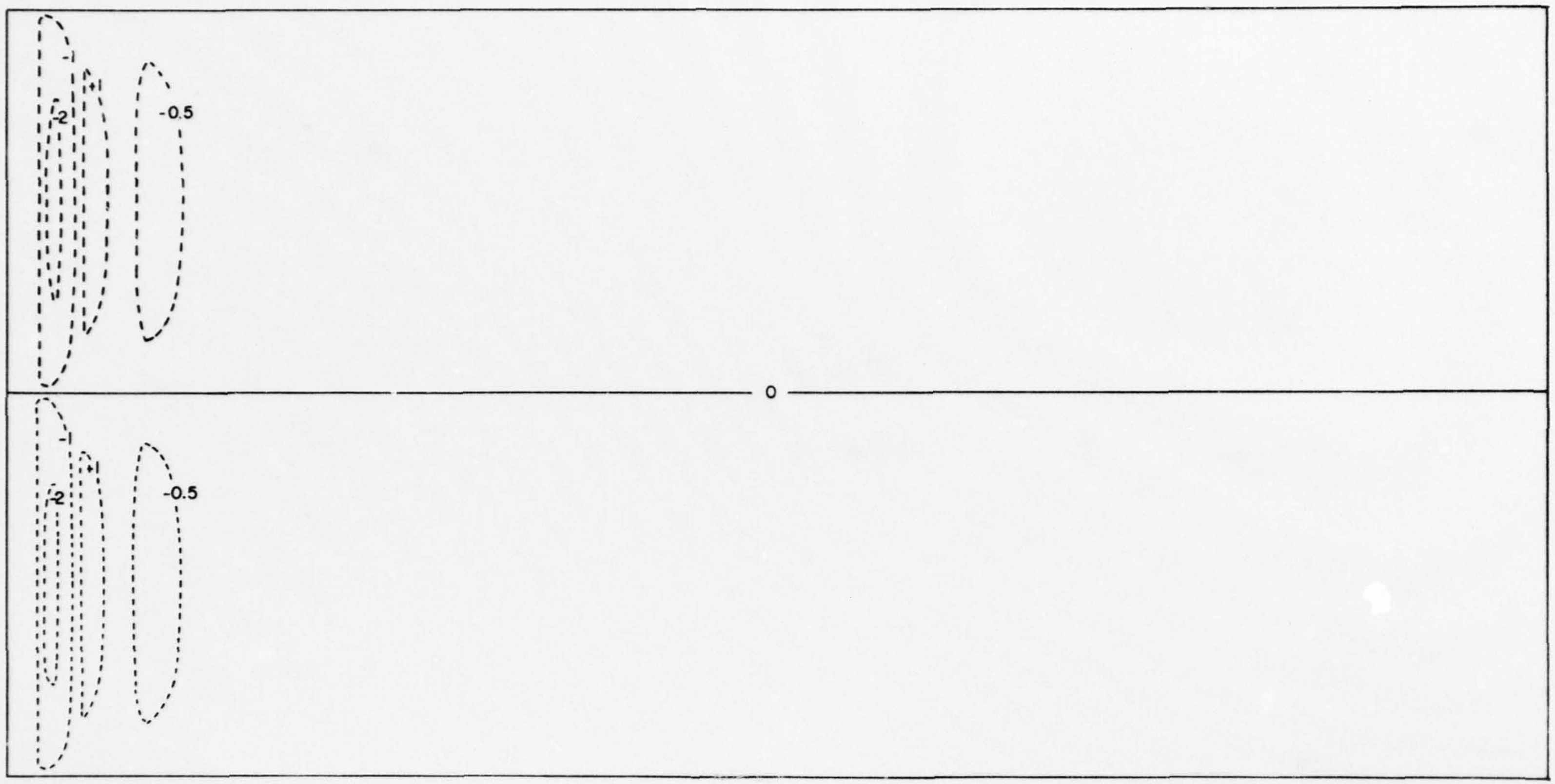

Figure 8. $\frac{1}{h}\int_{o}^{h} u\,dz \cdot \frac{\partial}{\partial x}\int_{o}^{h} v\,dz$ in c.g.s. units $\times 10^{-4}$

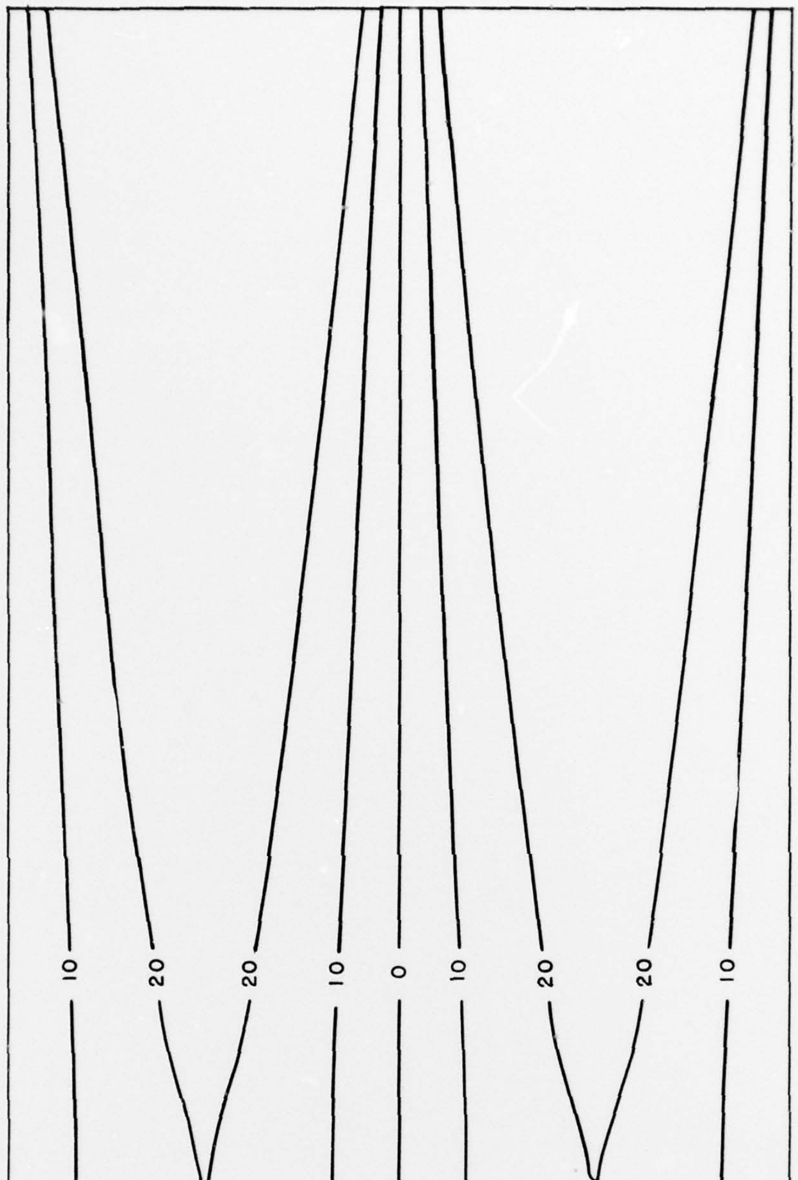

Figure 9. $\int_0^h v \frac{\partial v}{\partial y} \, dz$ in c.g.s. units X 10^{-4}.

Figure 10. $\frac{1}{h} \int_{o}^{h} v\,dz \cdot \frac{\partial}{\partial y} \int_{o}^{h} v\,dz$ in c.g.s. units $\times 10^{-4}$